Homework 3: Electrostatics
Due Monday, September 18

Important: You may not use MATHEMATICA or similar software on this assignment. This includes Maple, Matlab, Wolfram Alpha, or any other system that performs integrals for you. Show enough work to convince me that you have worked out the integrals.

Problem 1: Electric field due to charged square sheet

Find the electric field at a height z directly above ($z > 0$) the center of a square sheet with sides of length L and a uniform surface charge σ. Check your result for the limiting cases $z \ll L$ and $z \gg L$.

> HINT: "Check your result" means that your answer should reduce to something more familiar in those two cases. When $z \ll L$ it should look like the electric field for an infinite charged sheet. When $z \gg L$ it should look approximately like the electric field for a point charge $q = L^2\sigma$.

Problem 2: Charge density for a point charge

Suppose there is a point charge q located at $\vec{r}\,'$. Determine the volume charge density $\rho(\vec{r})$ for this point charge by applying the differential form of Gauss's law to its electric field:

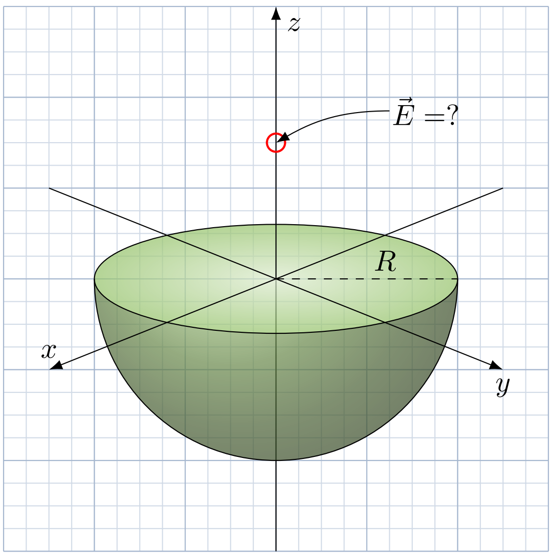
$$\vec{E}(\vec{r}) = \frac{q}{4\pi\epsilon_0}\frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} = \frac{q}{4\pi\epsilon_0}\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}\ . \tag{1}$$

Problem 3: Electric field due to a charged hemisphere

A hemisphere with radius R has a constant volume charge density ρ. Find the electric field at a point on the hemisphere's axis of symmetry, above the flat side. To be a little more specific, let's set up our coordinates so the hemisphere is

$$x^2 + y^2 + z^2 \leq R^2 \quad \text{with} \quad z \leq 0\ . \tag{2}$$

Find the electric field at a point on the z-axis with $z > 0$.



> HINT: Use a combination of spherical polar coordinates (r, θ, ϕ) and Cartesian unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$ when setting up and evaluating the integral.

Problem 4: Electric field due to a charged line segment

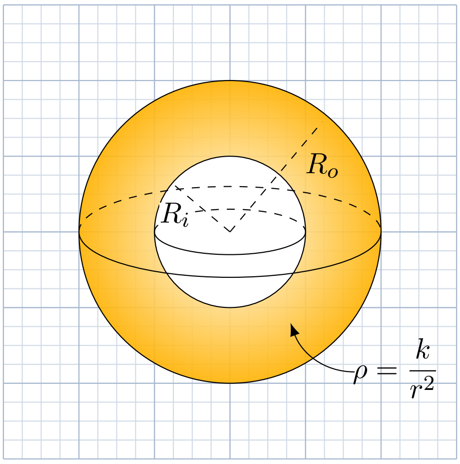
A straight line segment $-L/2 \leq x \leq L/2$ carries a uniform (constant) line charge λ. Find the electric field at an arbitrary point $\vec{r} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$. What happens to your result if you let $L \to \infty$, while keeping λ constant?

Problem 5: Electric field due to charged spherical shell

A hollow spherical shell with inner radius R_i and outer radius R_o carries a volume charge density in the region $R_i \leq r \leq R_o$

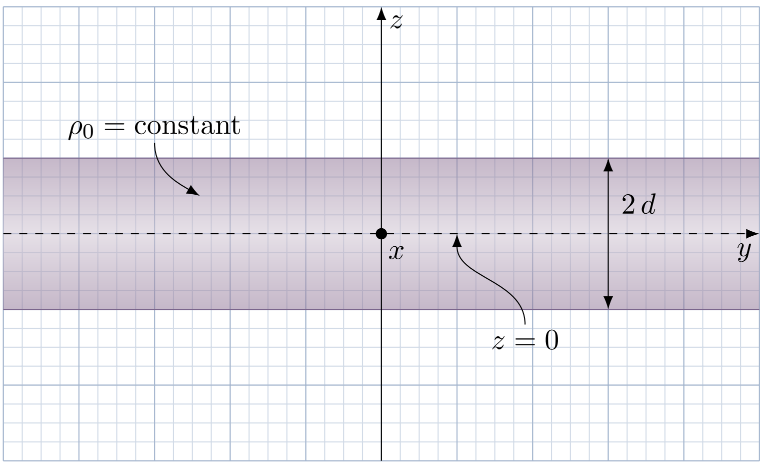
$$\rho = \frac{k}{r^2} \ , \tag{3}$$

where k is a positive number with appropriate units. Use Gauss's Law to find the electric field in the three regions (i) $r < R_i$, (ii) $R_i \leq r \leq R_0$, (iii) $r > R_o$, and then sketch a plot of $|\vec{E}|$ as a function of r.



Problem 6: Electric field due to an infinite plane slab

An infinite plane slab of thickness $2d$ carries a uniform (constant) volume charge density ρ_0. Find the electric field as a function of z, with $z = 0$ (the x-y plane) at the center of the slab. Plot the z-component of the electric field as a function of z. Remember to include both positive and negative values of z in your plot. You can assume that the charge density ρ_0 is positive.[1]



[1]Putting a subscript zero on a quantity to indicate that it is a constant, rather than a function, is a common notation.